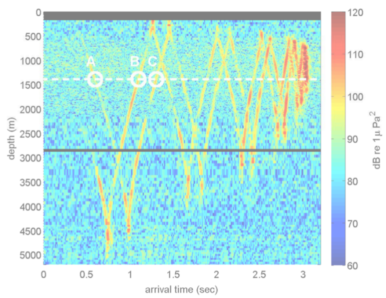
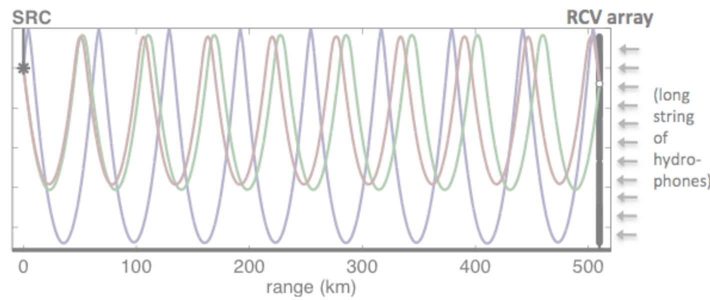
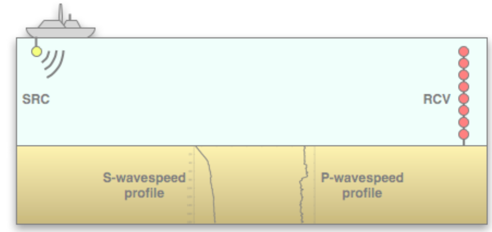






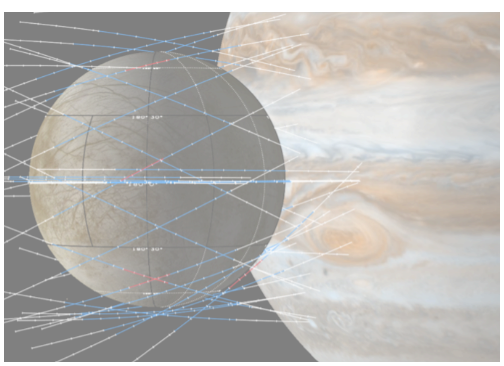
The use of inverse problems in geophysics for remote sensing with acoustics, electromagnetics, and gravimetry

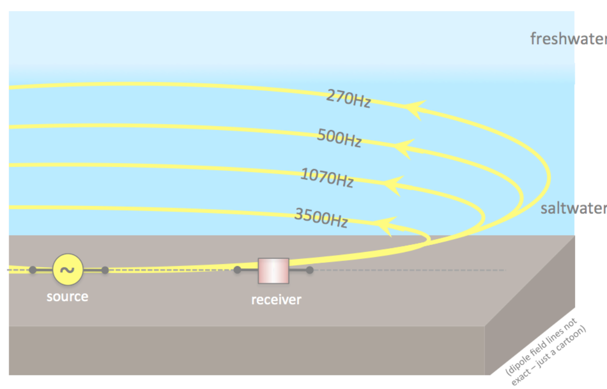
Andrew Ganse, Applied Physics Laboratory, UW

presented to Undergraduate Mathematical Sciences Seminar,

Applied Computational and Mathematical Sciences program, University of Washington

5 March 2015





A little bit about me

Home:

Work:

“Jim! I’m a geophysicist, not a mathematician!”

(R.I.P. Leonard Nimoy…)

School:

UWEE
undergrad :
BS electrical
engineering

Worked at APL for years before returning to school…

UW-ESS

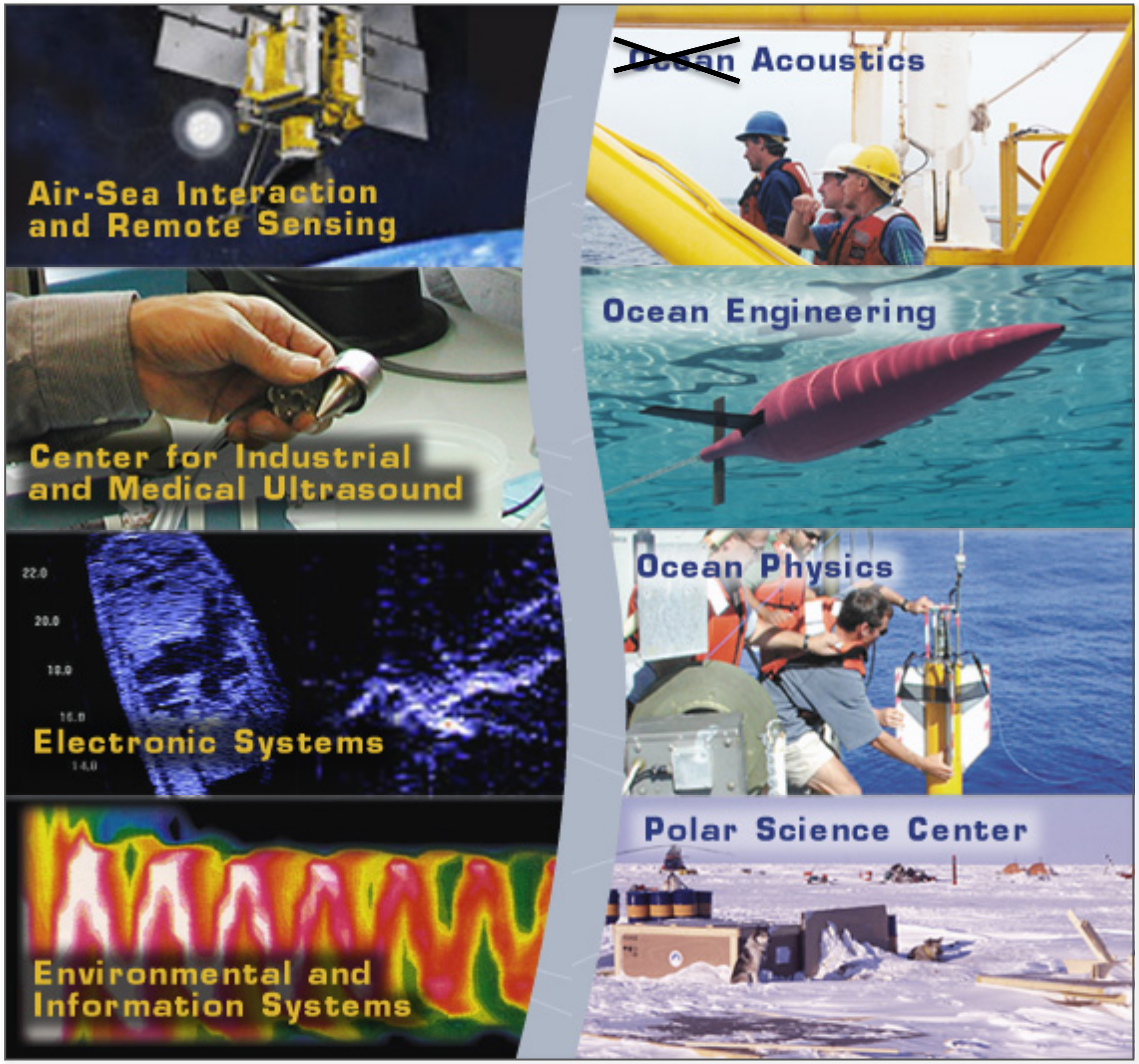
PhD geophysics
almost 20 years after BS!

A little bit about APL-UW

Applied Physics Laboratory, University of Washington

A little bit about APL-UW

Departments within the Applied Physics Laboratory



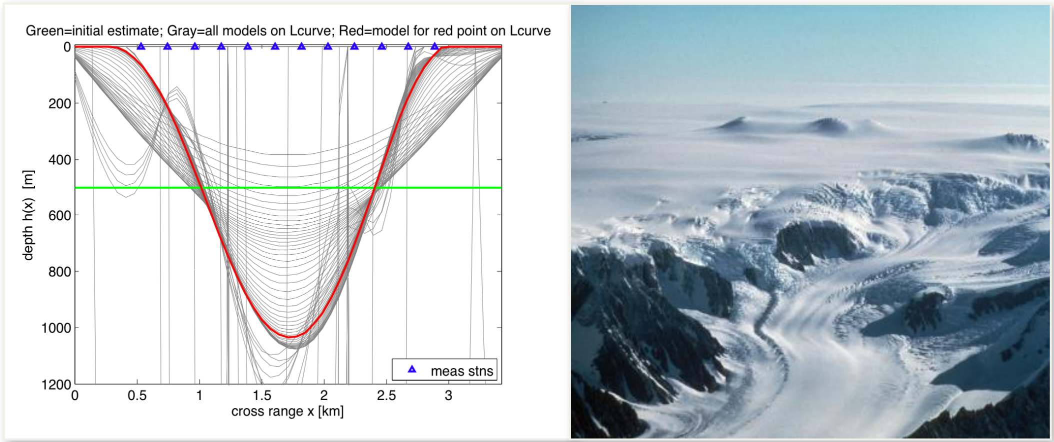
+ Support Depts:

Administrative
Accounting/Contracts
Building Maintenance
Graphics
Multimedia
Library
Machine Shop
Wood Shop
Electronic Shops
Shipping/Receiving
Security

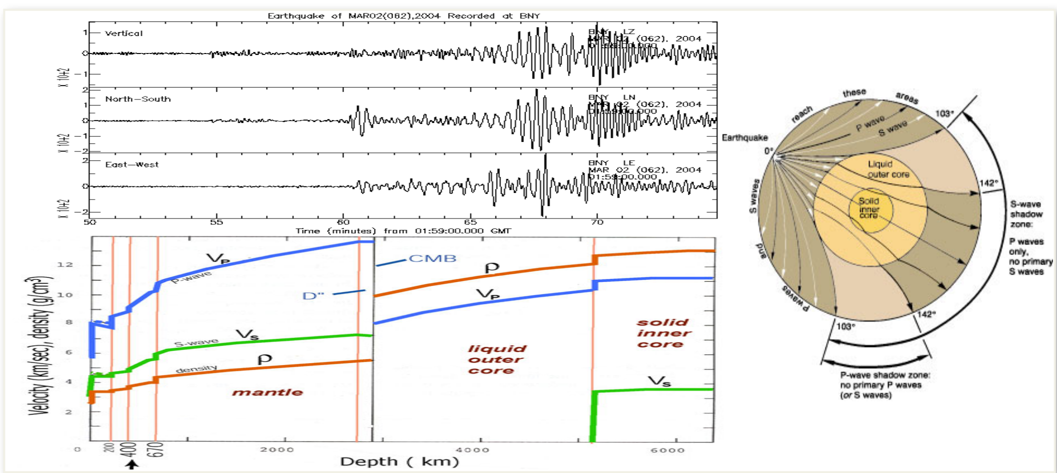
...and more...

Intro to inverse theory via examples

Glacier gravimetry: estimate glacier cross-section from gravity measurements



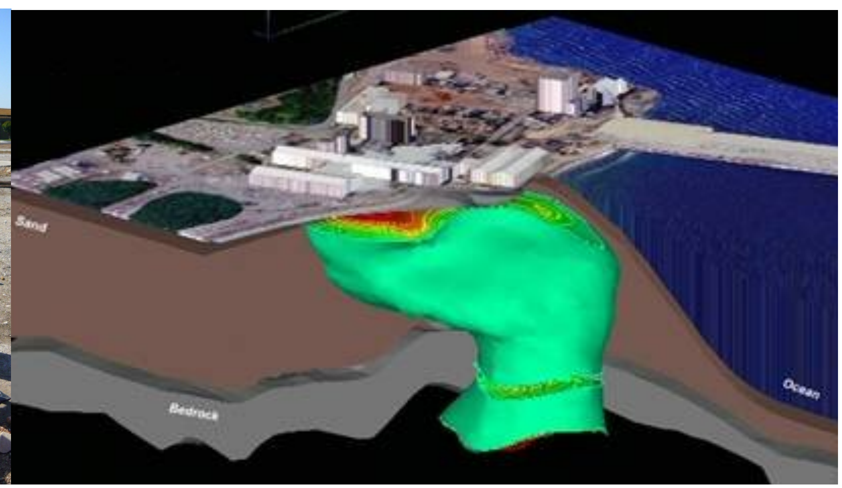
Global seismic inversion: estimate Earth's interior wavespeeds & densities from EQ seismograms



Intro to inverse theory via examples

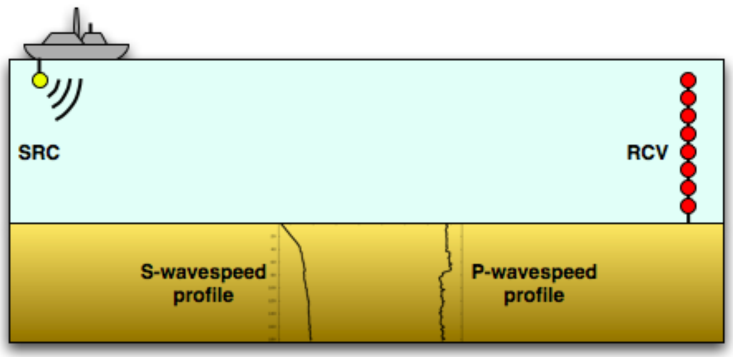
Computerized Tomography (CT) scans: estimate 3D body interior densities from Xray atten

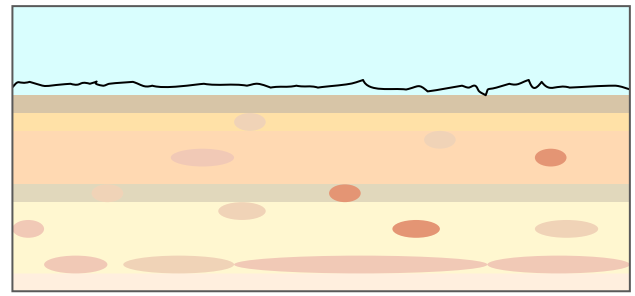
Groundwater contamination: estimate source leakage function from groundwater samplings



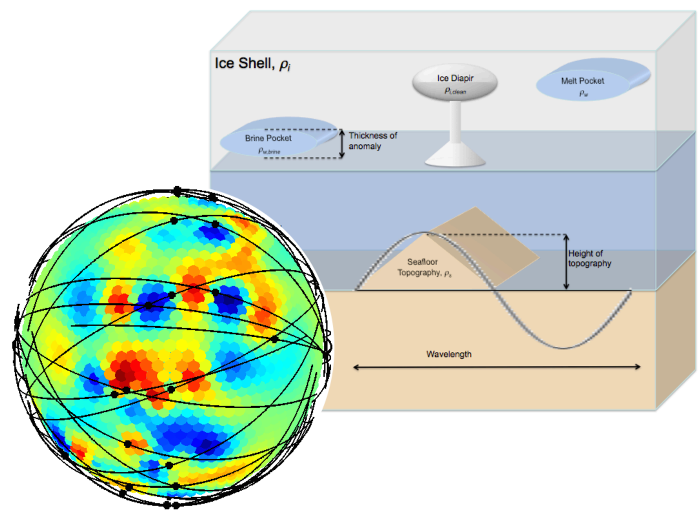
Intro to inverse theory via examples

Ocean bottom ("geoacoustic") inversion: estimate seafloor properties from sonar in water





Radio doppler gravimetry of planetary bodies: estimate density of icy moon interiors



Deduction vs. Induction

Note the common theme in those examples (but there are others):
inferring properties of interior from measurements on an exterior.

```
predicted_data  =  somefunction(  model_of_interest  )
```

gravity(x_i)
traveltime($depth_i$)
waveintensity(x_i,t_j)
dopplerfreq(t_j)
chemconcentration($x_{i,}t_j$)

density(x,z)
wavespeed(z)
temperature(z,t)
chemsrcleakage(t)
etc…

deduction

"forward problem"

effect

observed measurements

cause

e.g. ground structure

induction

"inverse problem"

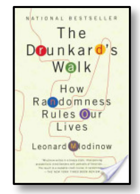
Probability vs. Statistics

Again the difference between deduction and induction

In these cases it is the latter scenario that is more often useful in life: outside situations involving gambling, we are not normally provided with theoretical knowledge of the odds but rather must estimate them after making a series of observations. Scientists, too, find themselves in this position: they do not generally seek to know, given the value of a physical quantity, the probability that a measurement will come out one way or another but instead seek to discern the true value of a physical quantity, given a set of measurements.

I have stressed this distinction because it is an important one. It defines the fundamental difference between probability and statistics: the former concerns predictions based on fixed probabilities; the latter concerns the inference of those probabilities based on observed data.

- Leonard Mlodinow
The Drunkard's Walk
(highly recommended!)



deduction

←

"forward problem"

effect

observed measurements

cause

e.g. ground structure

induction

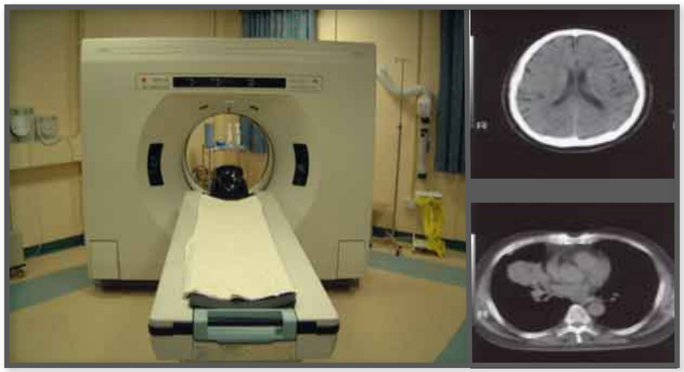
→

"inverse problem"

Analytical inversion

CT scans and the Inverse Radon Transform

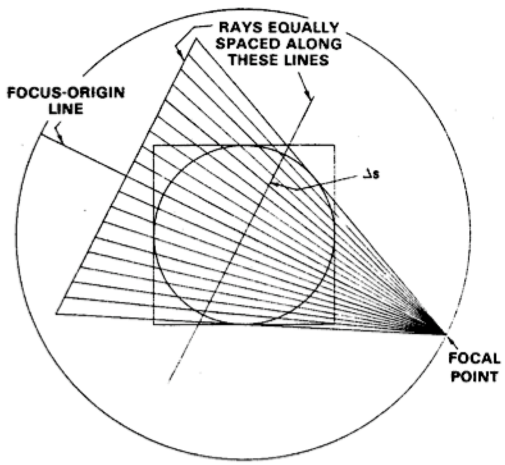
*Sortof an exception here – most real-world problems are too **UNSTABLE** for this approach…*

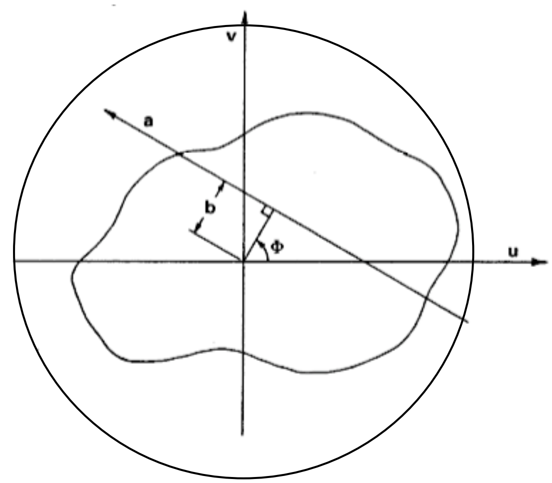


Body densities $x(.)$ at internal locations;
X-ray attenuation measurements $y(.)$
at locations around the circular perimeter:

$$y(b,\phi) = \int_{-\infty}^{\infty} x(a,b,\phi)\,da$$

$$x(u,v) = \int_{0}^{2\pi}\int_{-\infty}^{\infty} f\left[y(b,\phi), b, \phi, u, v\right] db\, d\phi$$





Rohler & Krishnaprasad, 1980

Model space vs. data space

- We wish to invert an integral equation :

$$d(s) = \int g(s,x)m(x)dx \longrightarrow m(x) = \ldots\textbf{?}$$

("Fredholm integral equation of the second kind")

$m(x) =$ the "model" (e.g. as in "Earth model")

$d(s) =$ the "data" (measurements)

$g(s,x) =$ the "kernel" function

Most real-world problems are too ***UNSTABLE*** *for analytical approach…*
What does that mean?

Stability

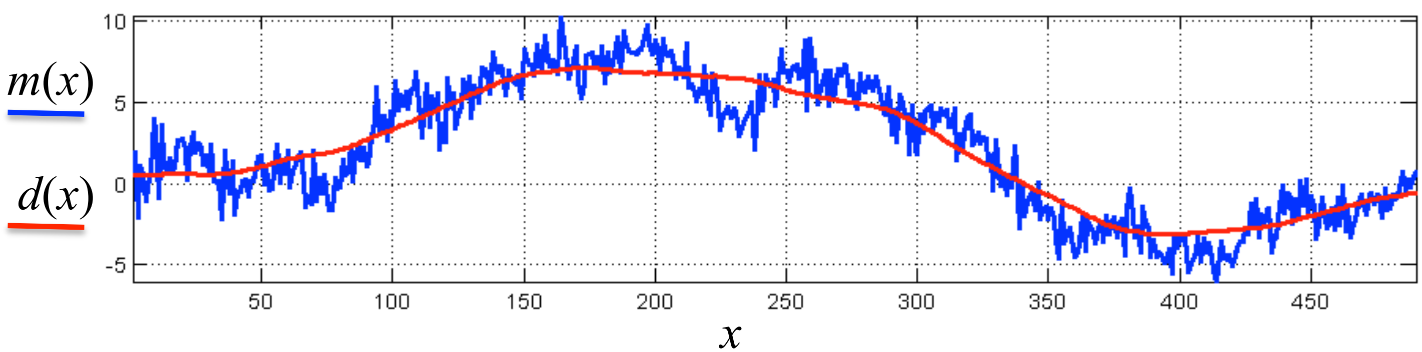
- We wish to invert an integral equation :

$$d(s) = \int g(s,x)m(x)dx \longrightarrow m(x) = \ldots ?$$

("Fredholm integral equation of the second kind")

Simple special case where data space is same as model space – curve fitting: $s = x$

A smoothing mechanism : think of $g(x,x)$ as boxcar functions → running average



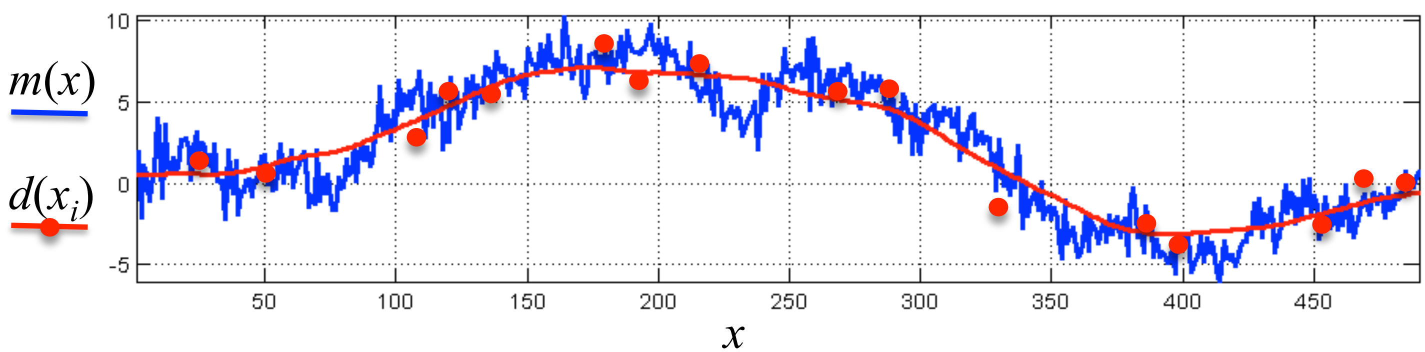
Big fluctuations in m(x) get turned into small fluctuations in d(s).
In the inverse, small fluctuations in d(s) get turned into big fluctuations in m(x).

Uncertainty

Noise on the data maps into uncertainty in the estimated model.

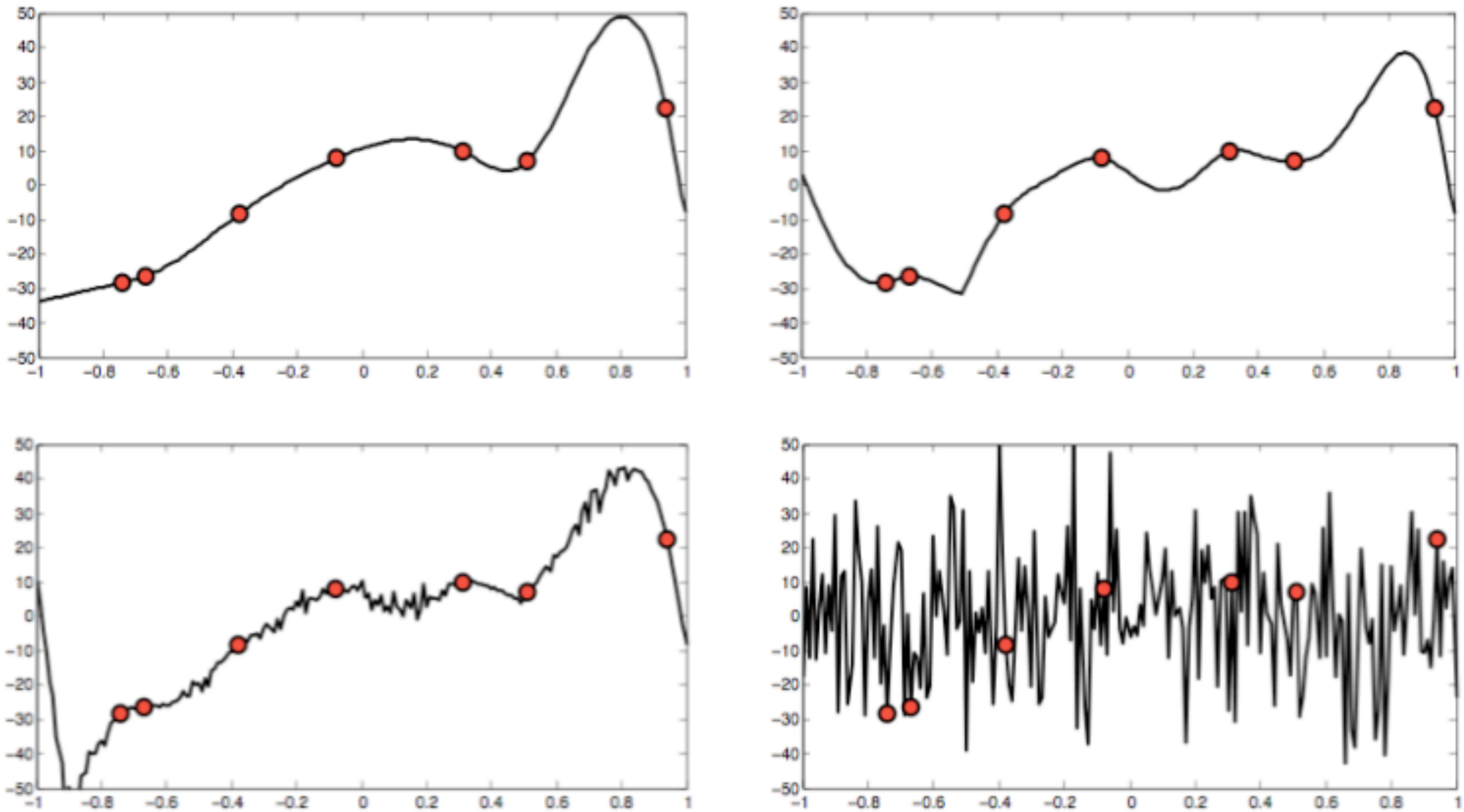
$$d(s) = \int g(s,x)m(x)dx \longrightarrow d(s_i) = \int g(s_i,x)\, m(x)dx + \epsilon(s_i)$$

A smoothing mechanism : think of $g(x,x)$ as boxcar functions → running average



Ganse

Non-uniqueness



uh-oh, infinitely many curves produce predictions that fit the same data to within the noise

Inversion via optimization/parameter estimation

Geophysical problem :

$$d_i = \int g_i(x)\, m(x) dx + \epsilon_i$$

Parameterize m(x) to turn the inverse problem into a parameter estimation :

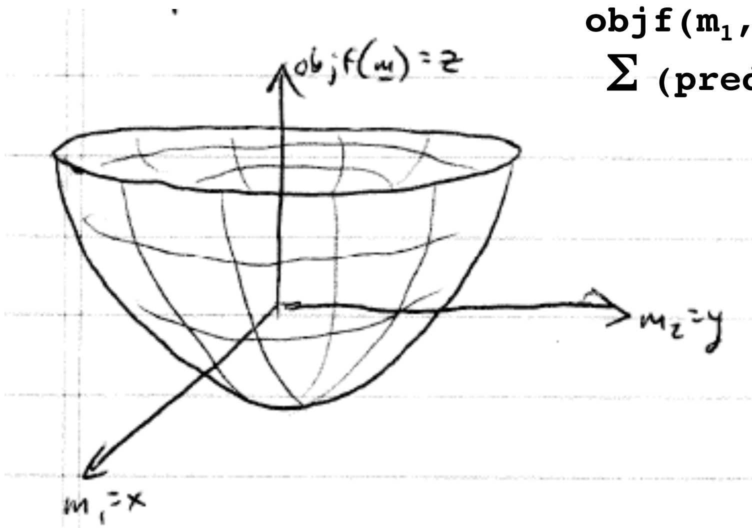
$$m(x) = \sum_j m_j\, b_j(x) \longrightarrow d_i = G_{ij}\, m_j + \epsilon_i$$

(but still must worry about existence, stability, uniqueness, and uncertainty)

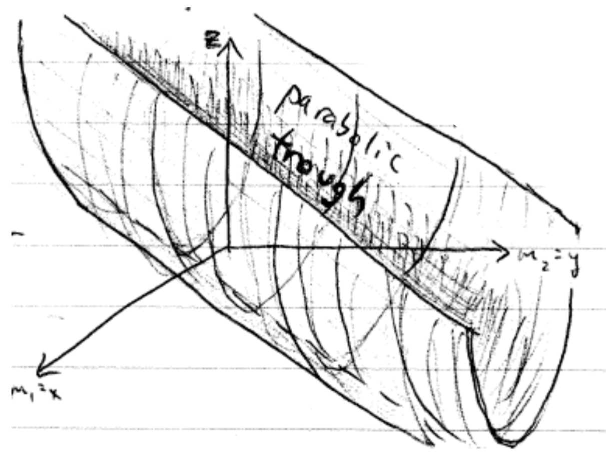
Approach: instead of trying to derive `somefunction`$^{-1}$`(model)`
from `predicted_data = somefunction(model)`*,*
we tweak `model` *until* `predicted_data` *is close to measured data*

Rank deficiency and ill-posedness

Define "objective function" *objf*(**m**) as a distance between the measured and modeled data. (i.e. sum of squares of differences)



$$\texttt{objf}(m_1,m_2,\ldots) = \sum \texttt{(predicted\_data}(m_1,m_2,\ldots) - \texttt{measured\_data)}^2$$



Need for regularization – adding information – via model constraints or prior probabilities

Frequentist vs. Bayesian probability

A debate raging for 200+ years in the prob/stats community!

- **Frequentists** define probability in terms of frequency of repeatable events.
  So one can't know anything about model before the event/experiment.
  Regularization takes form of model constraints, so not solving same problem as you started with (e.g. solving for a smooth version of true model).

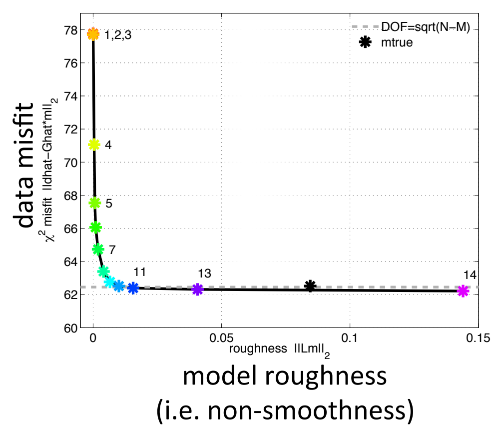
- **Bayesians** define probability in terms of degree of belief.
  So one *can* know about the model before the event/experiement.
  Regularization takes form of prior probabilities for the model parameters;
  so you ARE solving same problem started with, maybe get prior via other meas.

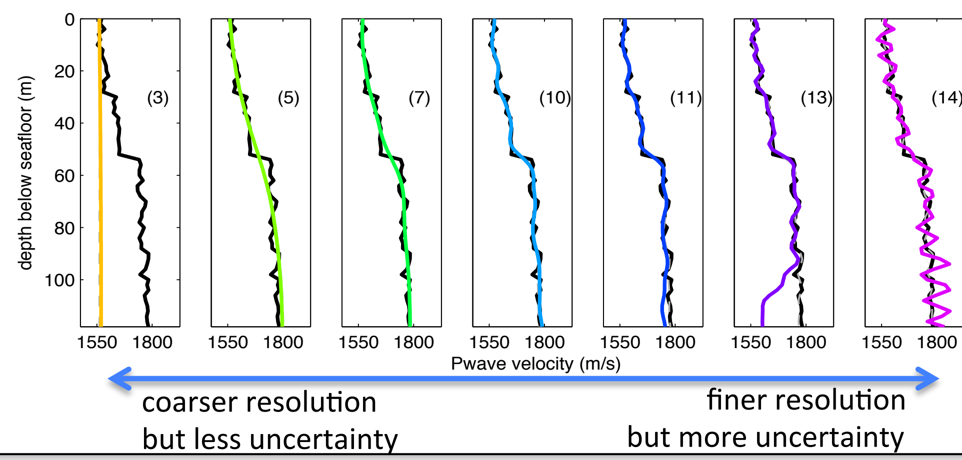
Frequentist inverse problem concepts

- A common approach is solving some smoothed problem – so it's not same problem you started with – then stating amount of smoothing with the results. But key is data/noise can *automatically* determine the amount of smoothing.

- Occam's Razor rationale – this results in fewest number of features in model that aren't required by the data.

- Trade-off between uncertainty & resolution of model solution – no free lunch:





Bayesian inverse problem concepts

- Bayes' Rule: (from definition of conditional probability)

$$p(\mathbf{m}|\mathbf{d}) = \frac{p(\mathbf{d}|\mathbf{m})\, p(\mathbf{m})}{p(\mathbf{d})} \propto p(\mathbf{d}|\mathbf{m})\, p(\mathbf{m})$$

"posterior" distribution of model parameters

"data likelihood" function

"prior" distribution of model parameters

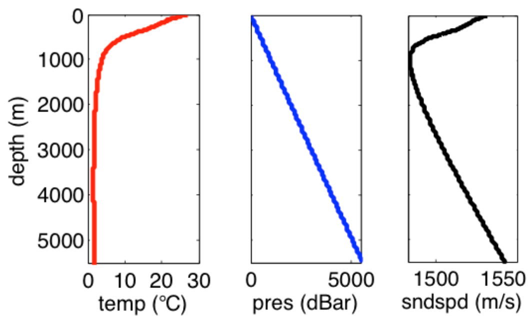
- End result is a probability density function of model parameters.

- Unlike frequentist case, no uncertainty/resolution trade-off here – the prior PDF is simply updated to the posterior PDF using the information in the data.

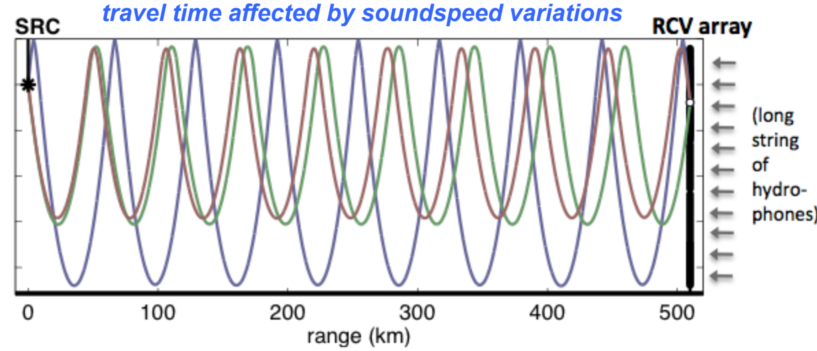
Case #1. Acoustic oceanography / WPRM

(with Rex Andrew, Andrew White, Jim Mercer, APL-UW; & Worcester et al., SIO)

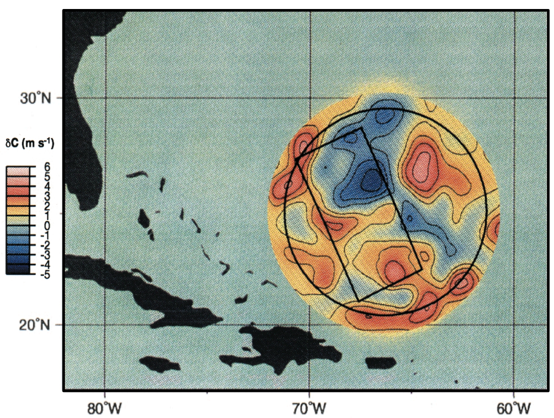
Ocean soundspeed mainly a function of temperature and pressure :



Long-range sound rays propagating in SOFAR waveguide – ***travel time affected by soundspeed variations***

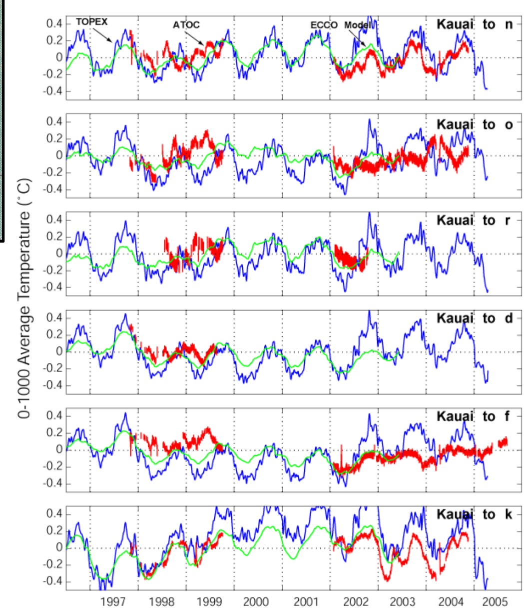


Solving for soundspeed variations

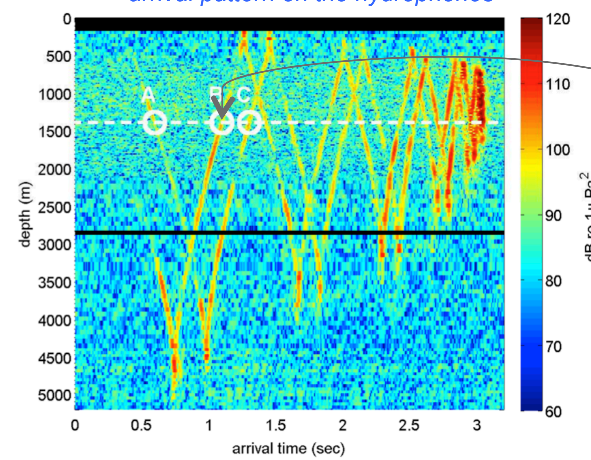


(Both these results by Brian Dushaw, APL-UW)

Monitoring ocean temperature variations via soundspeed variations over a decade…

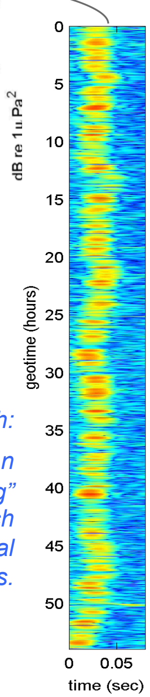


arrival pattern on the hydrophones



WPRM research:

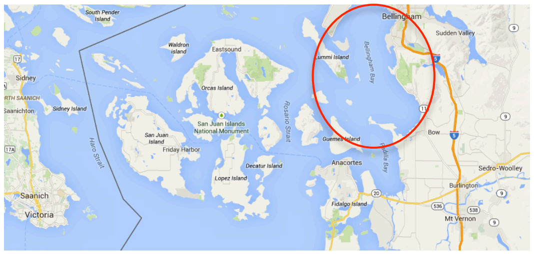
Measurements of ocean acoustic "twinkling" statistics do not match long-standing theoretical predictions.



#2. Estuarine salinity monitoring via EM

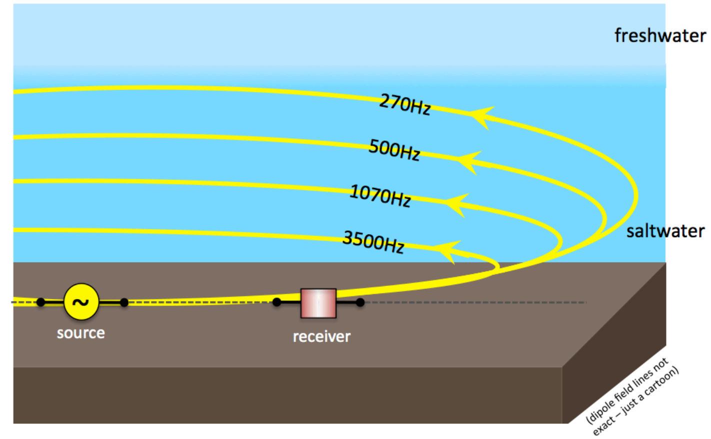
(with Tom Sanford & Zoli Szuts, APL-UW)

WA State's Bellingham & Samish Bays:
monitoring contaminated freshwater runoff onto shellfish habitats

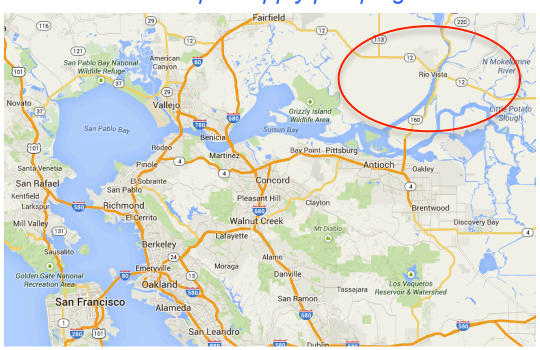


Physical layout –

E-fields at different frequencies have different skin depths, covering different spans of the water column. So invert G(f) to obtain σ(z).



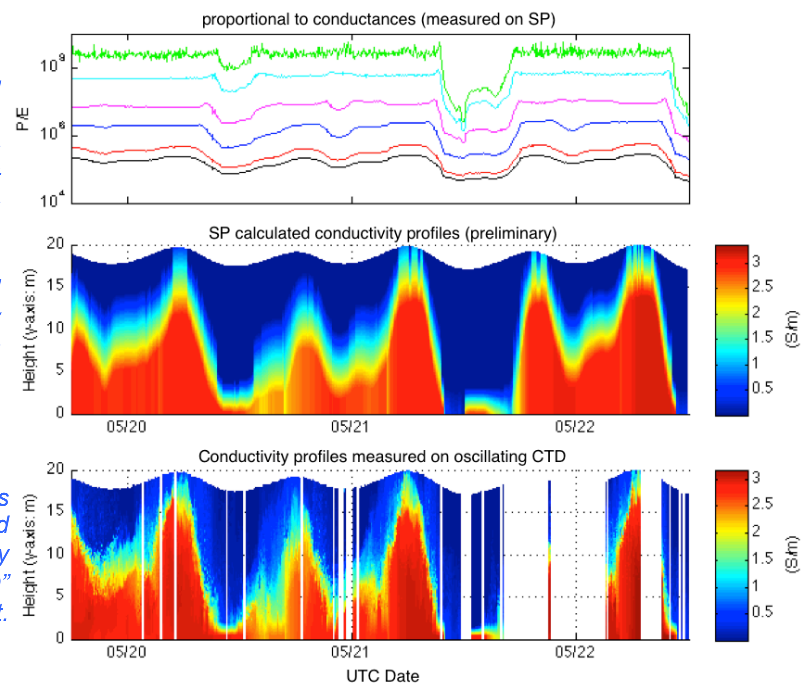
CA's Sacramento-San Joaquin River Delta:
monitoring tidal salt wedge intrusions re habitat/endangered-species restrictions on municipal supply pumping



Data: *E-field strength time-series at different frequencies*

Model: *Inverted electrical conductivity profiles as function of depth & time.*

Model as measured independently by a "CTD" instrument.

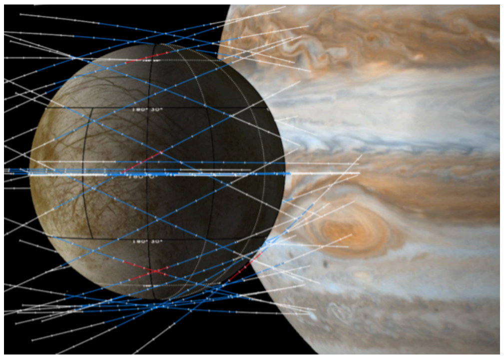


Ganse

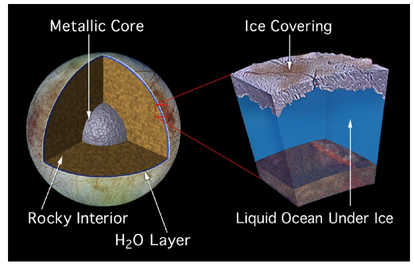
#3. Experiment-design/mission-planning for Europa Clipper gravimetry

(with Steve Vance, JPL & James Roberts JHU-APL)

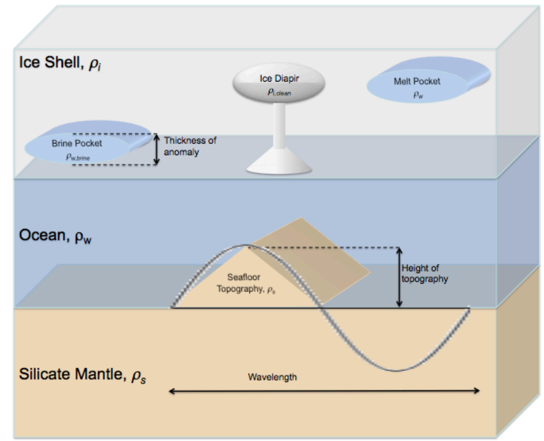
45 orbits in proposed Europa Clipper mission

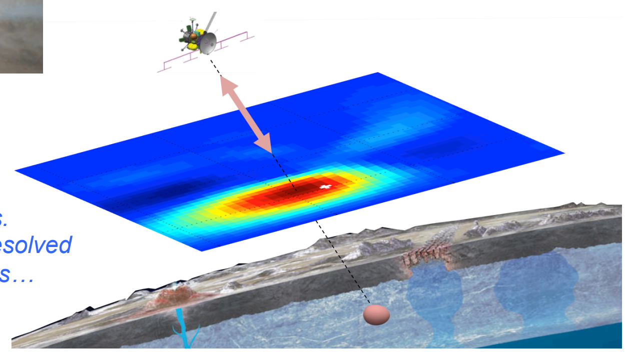


strong evidence for ocean/ice layer

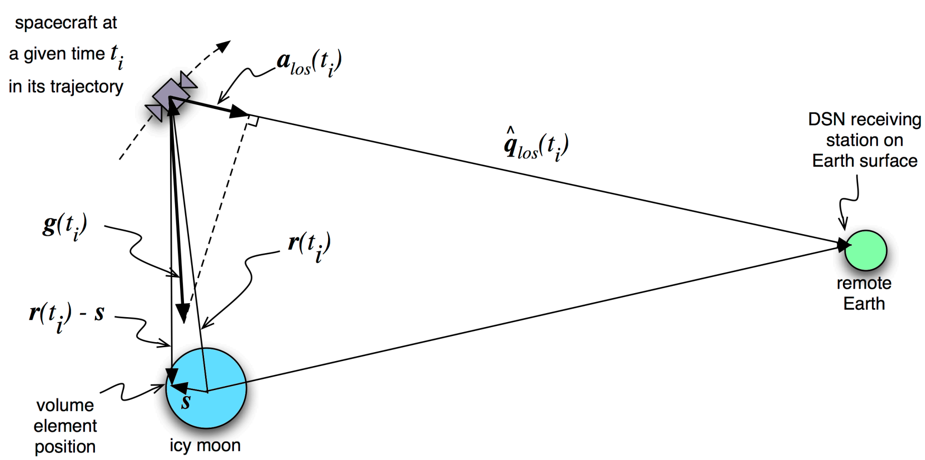


*Interest in H2O layer features like seamounts, diapirs, melt pockets – **what could Clipper resolve where?***

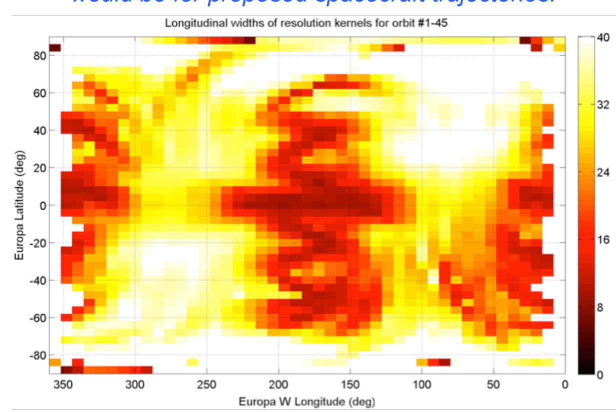




detection thresholds vs. what can actually be resolved from the measurements…



Rather than inverting data (no spacecraft yet!), here we estimate where the best quality inverse solutions would be for proposed spacecraft trajectories.



Inverse theory class: ESS 523

A graduate-level class, but open to undergrads (maybe senior year is best) without research requirement. Contact Professor Ken Creager, ESS-UW.

- **Overall**: learn how to do linear problems, then set up your nonlinear problem as a sequence of linear ones. (Note this presentation didn't discuss nonlinear problems.)

- Extensively uses **Matlab** or Octave (free/awesome GNU clone of Matlab) for a set of *wonderful* computational labs demonstrating key concepts. *Ok ok ok – I wrote the labs. ;-)*

- **Recommended Prerequisite background**:
  - Basic probability & statistics concepts -
    - e.g. mean, std dev, variance, covariance, correlation
  - Linear algebra -
    - e.g. matrix/vector arithmetic, transpose, inverse, null space, rank, condition number, eigenvalues/vectors, under/over-determined probs
  - Fourier transforms (time/space ←→ frequency)
  - Some idea of connection between the class and your research

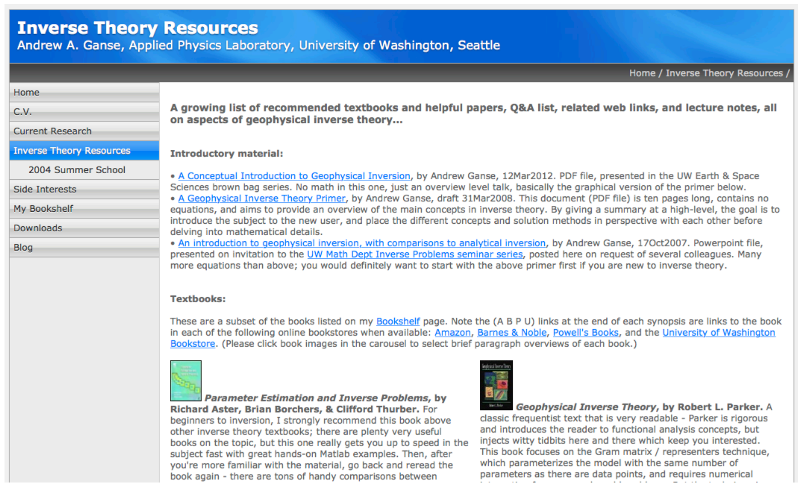
- No tests, but **weekly homework and labs, and a class project** based on your research

Shameless plug

http://staff.washington.edu/aganse

(also linked via APL directory)

Geophysical inverse theory educational resources


Inverse Theory Resources
Andrew A. Ganse, Applied Physics Laboratory, University of Washington, Seattle

Home / Inverse Theory Resources /

Home
C.V.
Current Research
Inverse Theory Resources
2004 Summer School
Side Interests
My Bookshelf
Downloads
Blog

A growing list of recommended textbooks and helpful papers, Q&A list, related web links, and lecture notes, all on aspects of geophysical inverse theory...

Introductory material:

• A Conceptual Introduction to Geophysical Inversion, by Andrew Ganse, 12Mar2012. PDF file, presented in the UW Earth & Space Sciences brown bag series. No math in this one, just an overview level talk, basically the graphical version of the primer below.
• A Geophysical Inverse Theory Primer, by Andrew Ganse, draft 31Mar2008. This document (PDF file) is ten pages long, contains no equations, and aims to provide an overview of the main concepts in inverse theory. By giving a summary at a high-level, the goal is to introduce the subject to the new user, and place the different concepts and solution methods in perspective with each other before delving into mathematical details.
• An introduction to geophysical inversion, with comparisons to analytical inversion, by Andrew Ganse, 17Oct2007. Powerpoint file, presented on invitation to the UW Math Dept Inverse Problems seminar series, posted here on request of several colleagues. Many more equations than above; you would definitely want to start with the above primer first if you are new to inverse theory.

Textbooks:

These are a subset of the books listed on my Bookshelf page. Note the (A B P U) links at the end of each synopsis are links to the book in each of the following online bookstores when available: Amazon, Barnes & Noble, Powell's Books, and the University of Washington Bookstore. (Please click book images in the carousel to select brief paragraph overviews of each book.)

***Parameter Estimation and Inverse Problems*, by Richard Aster, Brian Borchers, & Clifford Thurber.** For beginners to inversion, I strongly recommend this book above other inverse theory textbooks; there are plenty very useful books on the topic, but this one really gets you up to speed in the subject fast with great hands-on Matlab examples. Then, after you're more familiar with the material, go back and reread the book again - there are tons of handy comparisons between

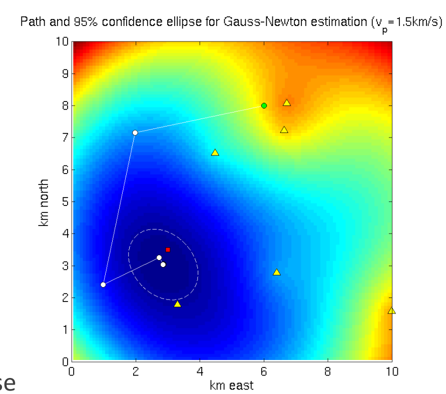
***Geophysical Inverse Theory*, by Robert L. Parker.** A classic frequentist text that is very readable - Parker is rigorous and introduces the reader to functional analysis concepts, but injects witty tidbits here and there which keep you interested. This book focuses on the Gram matrix / representers technique, which parameterizes the model with the same number of parameters as there are data points, and requires numerical

Introductory material, textbook summaries, recommended reading and links, teaching labs in process of being upgraded…

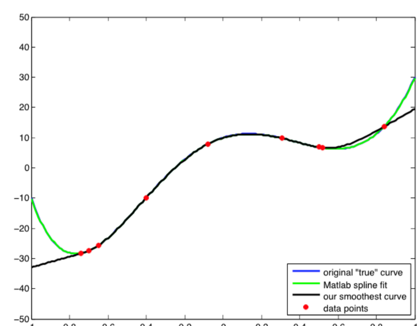
(This page is also linked from the Wikipedia "Inverse Problems" webpage…)

Some examples: Labs consist of lecture notes + technical/programming assignment (for which example code exists):

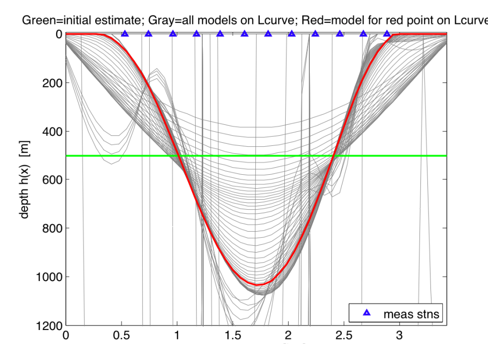
Labs 4 & 5: Parameter estimation of EQ source location, and objective surfaces



Lab 6: explore linear inversion with smoothing regularization via curve fitting, compare to cubic spline



New lab: adapting Parker's glacier gravimetry example (estimate bed interface)

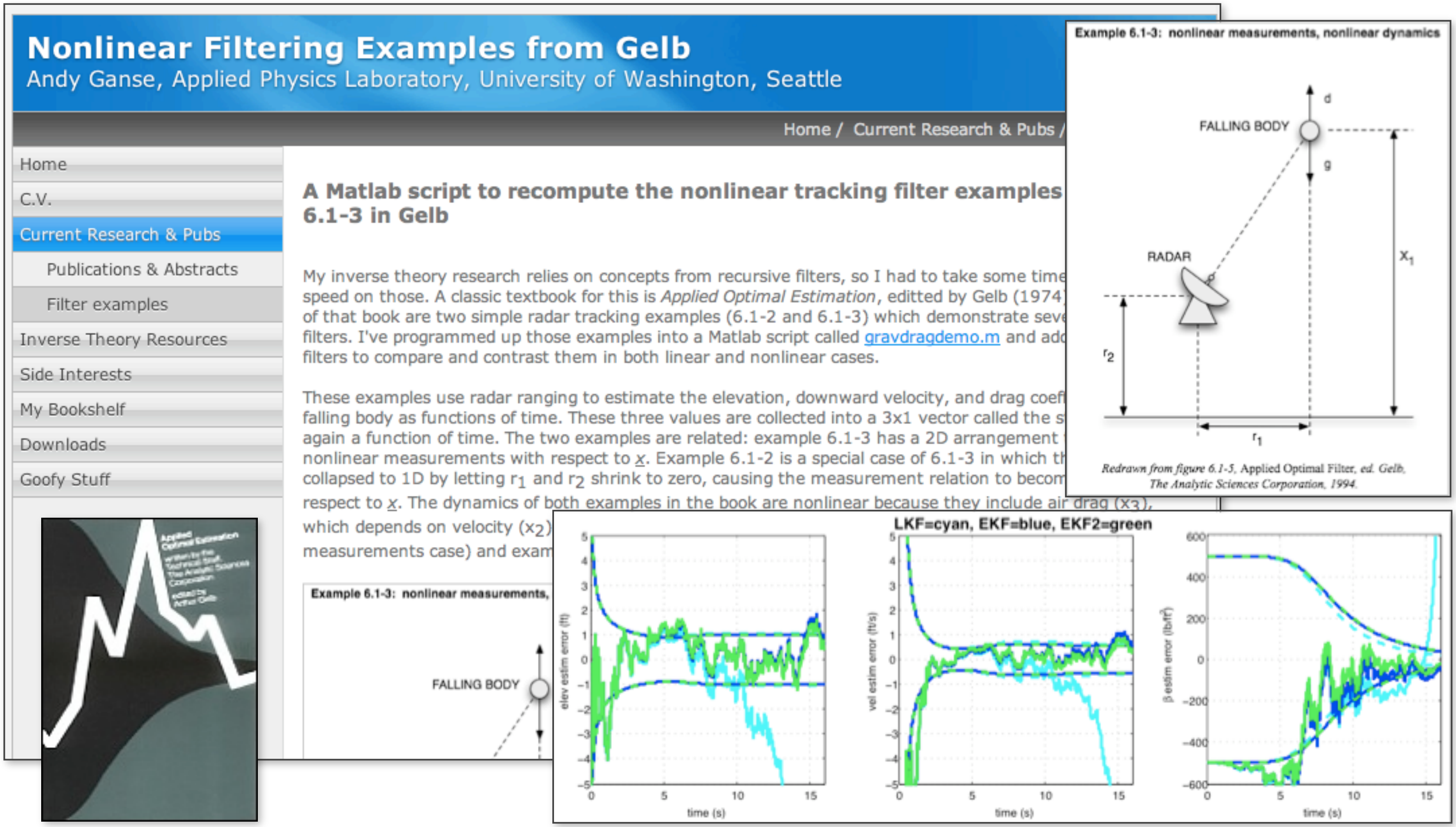


Another shameless plug

http://staff.washington.edu/aganse

(also linked via APL directory)

Tracking filters like Kalman filter are inverse problems with dynamics-based regularization

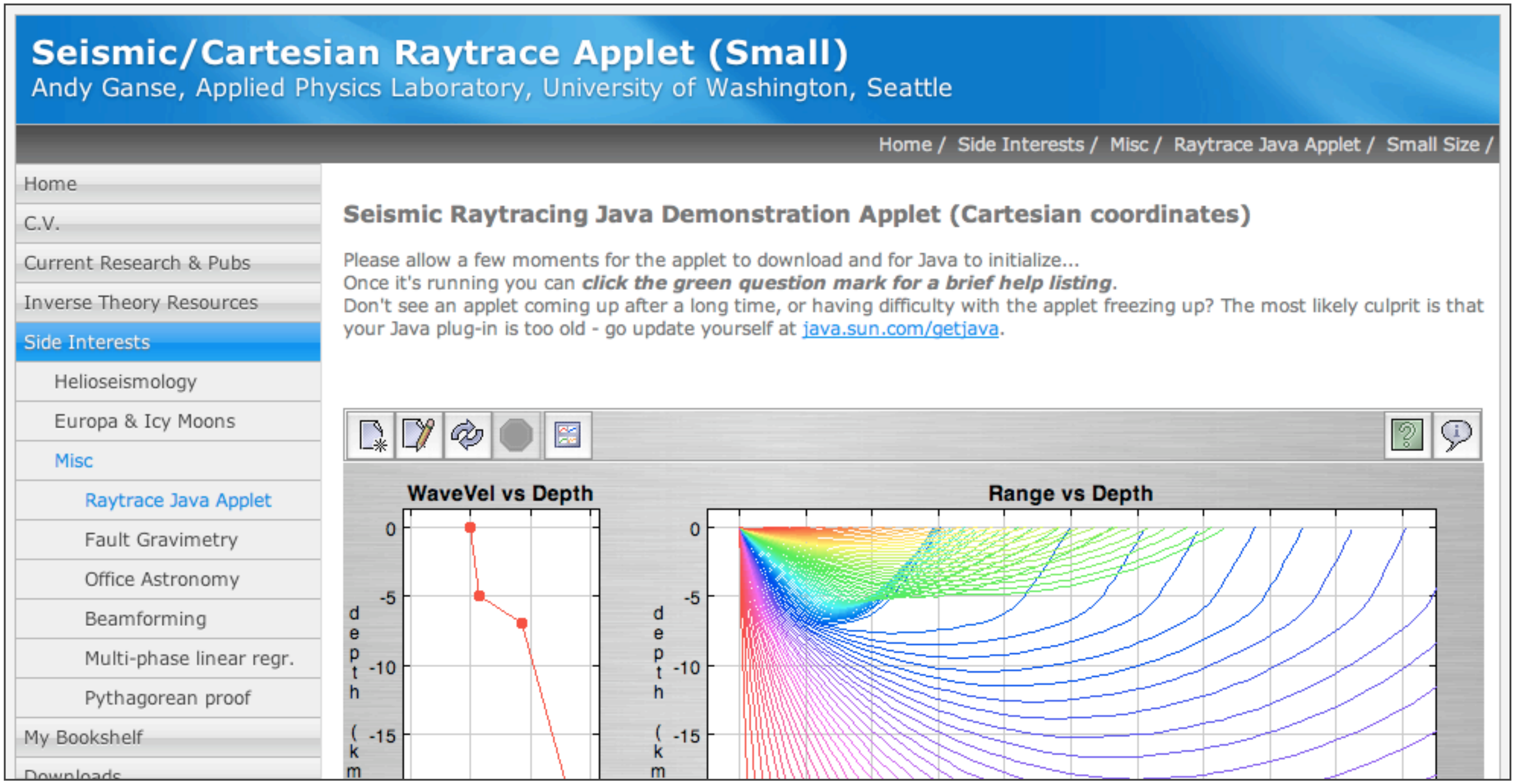


Fortunately, not too many shameless plugs…

http://staff.washington.edu/aganse

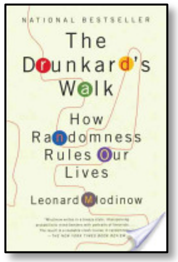
(also linked via APL directory)

Some of the wave propagation concepts I referred to are easily explored by playing with this

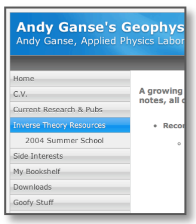


- Enter wave velocity profiles and watch the rays go!
- Spherical geometry one available too…

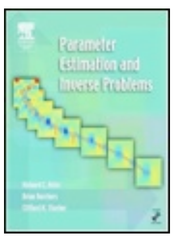
Recommended reading



Really fantastic popular book re probability and statistics:
The Drunkard's Walk,
by Leonard Mlodinow



My website (of course!) – pages on inverse theory resources, linear and nonlinear filter tutorial, ray-tracing, and much more.
http://staff.washington.edu/aganse



The best frequentist inverse theory textbook:
Parameter Estimation and Inverse Theory,
by Aster, Borchers, Thurber

The best Bayesian inverse theory textbook:
Inverse Problem Theory and Model Parameter Estimation,
by Albert Tarantola (available free online!)